# Anonymized Networks, Hidden Patterns, and Privacy Breaches

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Research on social networks: public vs. sensitive data

- Public data: Web pages, blogs, discussion boards, Wikipedia, open social networking sites.
- Sensitive: E-mail, IM, voice, physical proximity.
  E.g. nodes are e-mail or IM accounts; edge (v, w) if v communicates with w.

Anonymization of sensitive data:

- Consider research focused on structure and dynamics, not node identities.
- To anonymize: replace node names with random IDs.
- After doing this, is it safe to release?

With more detailed data, anonymization has run into trouble:

- Identifying on-line pseudonyms by textual analysis [Novak-Raghavan-Tomkins 2004]
- De-anonymizing Netflix ratings via time series [Narayanan-Shmatikov 2006]
- The AOL query logs ["This was a screw-up, and we're angry and upset about it." —AOL press release, 7 August 2006]

Our setting is much starker:

- No text, time-stamps, or node attributes
- Just a graph with nodes numbered 1, 2, 3, . . . , *n*.



Analogy with passive vs. active attacks in cryptography

- Passive attack: observe data as it is presented.
- Active attack: insert yourself into the process, potentially causing additional data to be generated.
- Template for an active attack on an anonymized network [Backstrom-Dwork-Kleinberg 2007]
  - Attacker can create (before the data is released)
    - nodes (e.g. by registering an e-mail account)
    - edges incident to these nodes (by sending mail)
  - Privacy breach: learning whether there is an edge between two existing nodes in the network.
  - Note: attacker's actions are completely "innocuous."

Main result: active attacks can easily compromise privacy by creating very few additional nodes.



Scenario:

Suppose a big company were going to release an anonymized communication graph on 100 million users.



An attacker chooses a small set of *b* user accounts to "target": Goal is to learn edge relations among them.



Before dataset is released:

- Create a small set of k new accounts, with links among them, forming a subgraph H.
- Attach this new subgraph H to targeted accounts.



When anonymized dataset is released, need to find H.

Why couldn't there be many copies of H in the dataset? (We don't even know what the network will look like ...) Why wouldn't it be computationally hard to find H?



In fact,

- Theorem: small random graphs H will likely be unique and efficiently findable.
- Erdös-Rényi construction; each edge present with prob. 1/2.



Once *H* is found:

Can easily find the targeted nodes by following edges from H.

First version of the attack:

- Create random H on (2 + ε) log n nodes.
  Can compromise ~ (log n)<sup>2</sup> targeted nodes.
- In experiments on 4.4 million-node LiveJournal graph, 7-node graph H can compromise 70 targeted nodes (and hence ~ 2400 edge relations).

Second version of the attack:

- Logarithmic size is not optimal.
- Can begin breaching privacy with H of size  $\sim \sqrt{\log n}$ .

Passive attacks:

• In LiveJournal graph: with reasonable probability, you and 6 of your friends chosen at random can carry out the first attack, compromising about 10 users.



- Random subgraph H (each edge with prob.  $\frac{1}{2}$ ).
- Link each targeted node to distinct subset of nodes in *H*.
- Must show
  - *H* is unique up to isomorphism (even after plugging it into rest of graph).
  - *H* is efficiently findable in unlabeled graph.
  - *H* has no internal symmetries (automorphisms); this is easy.



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# graph with no clique and no independent set of size $> 2 \log n$ .

 Quantitative bound for Ramsey's Theorem; one of the earliest uses of random graphs.

Basic calculation at the foundation of



- Build random *n*-node graph, include each edge with prob.  $\frac{1}{2}$ .
- There are  $< n^k$  sets of k nodes; each is a clique or independent set with probability  $\approx 2^{-k^2/2}$ .
- Product n<sup>k</sup> · 2<sup>-k<sup>2</sup>/2</sup> upper-bounds probability of any clique or indep. set; it drops below 1 once k exceeds ≈ 2 log n.

• Theorem (Erdös, 1947): There exists an *n*-node





#### Why is *H* Unique? Ideas from Ramsey Theory

Erdös: Graph is random, subgraph is non-random. Our case: Subgraph (H) is random, graph is non-random.

But main calculation starts from same premise:

Almost correct: there are < n<sup>k</sup> subgraphs that could be a second copy of *H*, and each is isomorphic to *H* with prob. ≈ 2<sup>-k<sup>2</sup>/2</sup>.



- Analysis is greatly complicated because
   *H* is plugged into full graph.
- New copies of *H* could partly overlap original copy of *H*.

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# Finding the subgraph H

To find *H*:

- Can assume there is a path through nodes 1, 2, ..., k.
- Start search at all possible nodes in G.
- Prune search path at depth j if edges back from node j don't match, or if degree of j doesn't match.



- Probability of a spurious path surviving to depth j is  $\approx 2^{-j^2/2}$  (modulo overlap worries).
- Overall size of search tree slightly more than linear in *n*.

# Experiments



k

- With 7 nodes, degrees in [20, 60], success rate > 90%.
- Average of 70 nodes compromised (2415 edges).
- Search tree about 90,000 nodes; recovery time < 1 sec.
- 7 nodes much less than 2 log n; randomization of degrees crucial to performance.

#### Stronger Theoretical Bound

- Variant on construction breaches privacy with H of size ~ √log n: Optimal up to constant factors.
- Construct *H* as before on *k* nodes, but connect to  $b = \frac{k}{3}$  targeted nodes.
- With high prob., min. internal cut in H exceeds b = cut to rest of graph.



# Stronger Theoretical Bound

Recovery:

- Break graph up along cuts of size ≤ b. Uses Gomory-Hu tree computation (e.g. Flake et al. 2004)
- Can prove that *H* will be one of the components after this decomposition.



Uniqueness of *H*:

- After breaking apart the graph, there are  $\leq \frac{n}{k}$  size-k components other than H.
- Each is isomorphic to H with probability  $\approx 2^{-k^2/2}$ .
- Now  $2^{-k^2/2}$  only has to cancel  $\frac{n}{k}$ , not  $n^k$ , so  $k \approx \sqrt{\log n}$  is enough.

- Recovery: Break graph up along cuts of size  $\leq b$ .
- To do this, build Gomory-Hu tree:
  - Tree *T* with same node set as original graph.
  - To find min. *v*-*w* cut in graph, delete min-weight edge on *v*-*w* path in *T*.



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- To find H: delete all edges in T of weight  $\leq b$ .
- Can prove *H* will be one of the resulting components.

Uniqueness of *H*:

- After breaking apart the graph, there are ≤ <sup>n</sup>/<sub>k</sub> components of size k, other than H.
- Each is isomorphic to H with probability  $\approx 2^{-k^2/2}$ .
- Now just need  $\frac{n}{k} \cdot 2^{-k^2/2} \ll 1$ , so  $k \approx \sqrt{\log n}$  is enough.



If you're already in the network, can you carry out this attack with no preparation?

- A node *v* recruits its neighbors.
- Suppose neighborhood subgraph N(v) is unique (and efficiently findable).
- If a node w is the only one to attach to a particular subset of N(v), then w is compromised.



What is the probability N(v) is unique, as a function of its size?

# Uniqueness of Neighborhood Subgraphs



In LiveJournal graph, number of distinct k-node N(v)'s:

• Small k: approx. the number of distinct k-node graphs.

• Larger k: approx. the number of nodes of degree k. If your degree is reasonably large, your pattern of friends is very likely unique.

#### **Passive Attacks**

- Don't need full neighbor subgraph.
- Attack has reasonable chance of success if you just recruit 4-6 of your friends.

- With 6 friends, can compromise about 10 nodes.
- Can compromise many more with some advance linking: a "semi-passive" attack.





What's the conclusion from all this?

- Doesn't apply to social network data that's already public; orthogonal to issues of legal/contractual safeguards.
- But widespread release of an anonymized social network? Danger: you don't what someone's hidden in there. And passive attacks don't even require advance planning.
- Further directions: privacy-preserving mechanisms for making social network data accessible.
  - May be difficult to obfuscate network effectively (e.g. [Dinur-Nissim 2003, Dwork-McSherry-Talwar 2007])
  - Interactive mechanisms for network data may be possible (e.g. [Dwork-McSherry-Nissim-Smith 2006])